Published by Institute of Physics Publishing for SISSA

Received: *April 30, 2007*
Revised: *July 10, 2007*
Accepted: *July 13, 2007*
Published: *July 19, 2007*

Superfield formulation of $4D$, $\mathcal{N}=1$ massless higher spin gauge field theory and supermatrix model

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Abstract: We study the relation between a supermatrix model and the free $4D$, $\mathcal{N}=1$ supersymmetric field theory of a massless supermultiplet with spins (3, 5/2). In order to do this, we construct a superfield formulation of the theory. We show that solutions of the equations of motion for the supermultiplet (3, 5/2) satisfy the equations of motion of a supermatrix model.

Keywords: Gauge Symmetry, Superspaces, M(atrix) Theories.

http://jhep.sissa.it/archive/papers/jhep072007057/jhep072007057.pdf

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1. Introduction

It has been known that there are problems in the construction of consistent interactions for massless higher spin gauge fields, though there are physically acceptable free field Lagrangian for them [1 – 4]. The Lagrangian can be obtained by postulating the gauge invariance which eliminates unphysical degrees of freedom [5]. A large amount of work has been done to construct interacting massless higher spin gauge field theories. Many attempts of them have encountered difficulties associated with requirements of gauge invariances [6 – 10], though there are some consistent interacting theories [11 – 13]. It is worthwhile to try to build interacting massless higher spin gauge field theories by a new approach.

In the previous paper [14], we have studied a matrix model as a new approach to formulate massless higher spin gauge field theory. As a first step towards constructing the theory, we have shown that the free equations of motion of bosonic massless higher spin gauge fields can be derived from those of the matrix model. This is based on a new interpretation of matrix models [15]. In [15], the authors have introduced a new interpretation of matrix models, in which matrices represent differential operators on a curved spacetime, and have shown that the vacuum Einstein equation can be obtained from the equations of motion of a matrix model. An advantage of this formalism is that the matrix model possesses gauge invariances manifestly, which are embedded in the unitary symmetry of the matrix model. Therefore it is interesting to analyze interacting massless higher spin gauge field theory using the matrix model.

In this paper, we study the relation between a supermatrix model and the free $4D$, $\mathcal{N} = 1$ supersymmetric field theory of massless supermultiplet with spins (3, 5/2) on the basis of [16]. In [16], the authors have extended the formalism in [15] to include supersymmetric field theories by replacing matrices by supermatrices in the matrix model. Furthermore, they have shown that solutions of the equations of motion for the $4D$, $\mathcal{N} = 1$ supergravity satisfy the equations of motion of the supermatix model. In this paper, we generalize their analysis to higher spin gauge field theory of a massless supermultiplet with spins (3, 5/2). In order to do this, we construct both on-shell and off-shell formulation for the free $4D$, $\mathcal{N} = 1$ supersymmetric field theory of a massless supermultiplet with spins (3, 5/2) in terms of superfields.[1] We show that solutions of the equations of motion for the supermultiplet satisfy the equations of motion of the supermatrix model. The formulations are quite similar to the superfield formulations of supergravity: In on-shell formulation, the equations of motion for the supermultiplet can be expressed as a constraint on field strengths. The superspace Bianchi identities subject to off-shell constraints are solved and superfield strengths are expressed by a set of superfields.

There are two ways to construct superfield formulations of supersymmetric field theories [17]: (1) One way is to study the off-shell representation to determine the linearized formulation in terms of constraint-free superfields and then construct covariant derivatives. (2) Another way is to start by postulating the existence of covariant derivative, and then determine what constraints they must satisfy and solve them in terms of a set of superfields. An off-shell superfield formulation of massless higher spin gauge field theory has been constructed [18] in the way (1). The formulation we construct in this paper is in the way (2).

There is another motivation for our study. Massless higher spin fields are expected to appear in the tensionless limit of string theory, since mass squared of them are all proportional to the string tension. On the other hand, matrix models are expected to be a nonperturbative formulation of string theory. Therefore our study may lead to further understanding of nonperturbative aspects of string theory.

The organization of this paper is as follows. In section 2, we briefly review the results of [16]. In section 3, we construct an on-shell formulation of a massless supermultiplet with spins (3, 5/2) in terms of superfield. In section 4, we study the relation between a supermatrix model and the superfield formulation of the supermultiplet. We show that solutions of the equations of motion for the supermultiplet satisfy the equations of motion of the supermatrix model. Section 5 is devoted to conclusions and future works. In appendix A, we summarize the on-shell constraints. In appendix B, we give the explicit forms of the

[1] In four dimensional spacetime, all higher spin fields can be described either by totally symmetric tensor or totally symmetric tensor-spinor fields. In this paper, we will restrict our consideration to four dimensional field theories.

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superspace Bianchi identities subject to the on-shell constraints. In appendix C, we give the results of the off-shell superfield formulation.

2. Supermatrix model

2.1 New interpretation of supermatrix model

In [15], a new interpretation of matrix models has been proposed in which matrices represent differential operators on a D dimensional curved space. Matrices act as *Endomorphisms* on a vector space, which means matrices map a vector space to itself. On the contrary, covariant derivatives map a tensor field of rank-n to a tensor field of rank-$(n+1)$. In order to interpret differential operators as matrices, we should prepare a vector space V which contain at least tensor fields of any rank. In [15], the authors have shown that such a space can be given by the space of functions on the principal Spin(D) bundle over a base manifold M. Furthermore, they have considered the large N reduced model of pure Yang-Mills theory as the matrix model. Applying this new interpretation to the matrix model, they have shown that the vacuum Einstein equation can be derived from the equations of motion of the matrix model.

However, supergravity cannot be embedded in the usual bosonic matrix model because there are no Grassmann variables in the matrix model. Thus, in order to describe supergravity by matrix models, we need to extend V to include Grassmann variables. It has been shown that this is implemented by extending manifold M to a supermanifold $\mathcal{M}$ and taking V to be the space of functions on the principal Spin(D) bundle over $\mathcal{M}$ [16]. In this extension, matrices are replaced by supermatrices, and covariant derivatives are replaced by supercovariant derivatives. In [16], the authors have considered the supermatrix model which is obtained by replacing matrices by supermatrices in the Large-N reduced model of pure Yang-Mills action,[2]

$$S = -\frac{1}{4}\mathrm{Str}\Big([\mathcal{A}_a, \mathcal{A}_b][\mathcal{A}^a, \mathcal{A}^b]\Big), \tag{2.2}$$

where $\mathcal{A}_a$ are hermitian and Grassmann even supermatrices with vector index. This action has SO(D) Lorentz symmetry and superunitary symmetry $U(N_e|N_o)$.[3] Applying the new

[2] We can consider the supersymmetric version of the supermatrix model, which is the supermatrix generalization of IIB matrix model [21],

$$S = -\frac{1}{4}\mathrm{Str}[\mathcal{A}_a, \mathcal{A}_b][\mathcal{A}^a, \mathcal{A}^b] + \frac{1}{2}\mathrm{Str}\overline{\Psi}\gamma^a[\mathcal{A}_a, \Psi], \tag{2.1}$$

where $\mathcal{A}_a$ are Grassmann even supermatrices and Ψ are Grassmann odd supermatrices. This action has global $\mathcal{N}=2$ supersymmetry, but we could not understand the meanings of global $\mathcal{N}=2$ supersymmetry of this model in the new interpretation. Thus, we restrict our consideration to (2.2).

[3] An even supermatrix $\mathcal{A}$ can be written as

$$\mathcal{A} = \begin{pmatrix} A_1 & B_1 \\ B_2 & A_2 \end{pmatrix}, \tag{2.3}$$

where A_1 are $N_e \times N_e$, A_2 are $N_o \times N_o$ bosonic matrices and B_1 are $N_o \times N_e$, B_2 are $N_e \times N_o$ fermionic matrices.

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interpretation to this supermatrix model, the authors have shown that solutions of the equations of motion for the $D=4$, $\mathcal{N}=1$ supergravity satisfy the equations of motion of the supermatrix model.

2.2 Massless higher spin fields

Let us see that there is a possibility that the supermatrix model involves the degrees of freedom of massless higher spin gauge fields. Before we begin discussing massless higher spin fields, we explain our notations. The coordinates of a superspace $\mathcal{M}$ are expressed as $z^M = (x^m, \theta^\mu)$, where $x^m (m = 1, \ldots, D)$ are bosonic and $\theta^\mu (\mu = 1, \ldots, D_s)$ are fermionic components. D_s is the dimension of spinor representation of Spin(D). Letters $M = (m, \mu)$ denote curved space indices and $A = (a, \alpha)$ denote local Lorentz indices. The supercovariant derivative ∇_A is defined as

$$\nabla_A = e_A{}^M(z)(\partial_M + \omega_M{}^{bc}(z)\mathcal{O}_{bc}), \tag{2.4}$$

where $e_A{}^M(z)$ is the supervielbein and $\omega_M{}^{bc}(z)$ is the superspin connection. Notice that ∇_A maps a rank-n tensor to a rank-$(n+1)$ tensor and $\mathcal{O}_{ab}$ acts on the local Lorentz indices of these tensors. Therefore we have

$$[\mathcal{O}_{ab}, \nabla_c] = \frac{1}{2}(\delta_{ac}\nabla_b - \delta_{bc}\nabla_a), \tag{2.5}$$

$$[\mathcal{O}_{ab}, \nabla_\alpha] = (\gamma_{ab})_\alpha{}^\beta \nabla_\beta, \tag{2.6}$$

in this setting, which will be used later.

Since each component of supermatrices $\mathcal{A}_a$ acts on the functions on the principal Spin(D) bundle over $\mathcal{M}$ as an *Endomorphism*, in general, $\mathcal{A}_a$ can be expanded as

$$\mathcal{A}_a = i\nabla_a + a_a(z) + \frac{i}{2}\{b_a{}^B(z), \nabla_B\} + \frac{i}{2}\{\omega_a{}^{bc}(z), \mathcal{O}_{bc}\} + \frac{i^2}{2}\{e_a{}^{BC}(z), \nabla_B\nabla_C\} + \cdots, \tag{2.7}$$

where i and anticommutator $\{\}$ are introduced to make $\mathcal{A}_a$ hermitian supermatrices.[4] Terms higher than first order with respect to the operators ∇_A and $\mathcal{O}_{ab}$ can be taken to be symmetric (or antisymmetric) under permutations of the operators, because antisymmetric (or symmetric) part can be absorbed in the term that is the lower order in ∇_A and $\mathcal{O}_{ab}$. We consider the expansion as a sum of homogeneous polynomials of ∇_A and $\mathcal{O}_{ab}$, whose coefficients are identified with massless higher spin gauge fields. Coefficients of the first order homogeneous polynomial will express gauge fields of the supermultiplet $(2, 3/2)$, and those of the second order one will express gauge fields of the supermultiplet $(3, 5/2)$ and so on. The number of independent components of higher spin gauge fields grows rapidly with degree in ∇_A and $\mathcal{O}_{ab}$.

[4] The formulation which has been given in [9] is similar to the one we use in this paper. In [9], the authors have introduced the differential one-form which is expanded in terms of the ordinary derivatives and the Lorentz generators. Thus, the components of the one-form are close to the matrices (2.7) in the case of flat spacetime. However, we can find two differences. One is that we have found the way to derive the equations of motion of massless higher spin fields. Another is that we have introduced the terms higher than second order in the Lorentz generators.

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If the supermatrix model has the degrees of freedom of massless higher spin fields, the gauge symmetries associated with those fields should be included. We find that the symmetries can be realized as the superunitary symmetry of the supermatrix model. Originally, the superunitary symmetry is written as

$$\delta \mathcal{A}_a = i[\Lambda, \mathcal{A}_a], \tag{2.8}$$

where Λ is a $N \times N$ hermitian supermatrix. In the new interpretation, Λ becomes a scalar operator expanded in terms of ∇_A and $\mathcal{O}_{ab}$.

Let us check how gauge transformations are generated by Λ in the case of the supermultiplet $(3, 5/2)$. In order to deal with this case, we need to keep track of the following terms

$$\mathcal{A}_a = i\nabla_a + \frac{(i)^2}{2} e_{a,}{}^{bc}(\nabla_b\nabla_c + \nabla_c\nabla_b) + \frac{(i)^2}{2} e_{a,}{}^{c\gamma}(\nabla_c\nabla_\gamma + \nabla_\gamma\nabla_c) + \cdots . \tag{2.9}$$

We take Λ as $\Lambda = \lambda^{c\gamma}(\nabla_c\nabla_\gamma + \nabla_\gamma\nabla_c)$, then (2.8) becomes

$$\delta \mathcal{A}_a = (\nabla_a \lambda^{c\gamma})(\nabla_c\nabla_\gamma + \nabla_\gamma\nabla_c) + \cdots . \tag{2.10}$$

Thus $e_{a,}{}^{c\gamma}$ transforms as

$$\delta e_{a,}{}^{c\gamma} = \nabla_a \lambda^{c\gamma} + \cdots . \tag{2.11}$$

This can be considered as the supergauge transformation for the spin-5/2 field.

2.3 Superfield formulation

In order to study the relation between the supermatrix model and supersymmetric field theories of massless higher spin supermultiplets, we should compare the equations of motion of the supermatrix model with those of the supermultiplets. Since the local fields which appear in (2.7) live in superspace, the equations of motion of the supermatrix model are written in terms of superfields. Thus, we should write the equations of motion of massless higher spin supermultiplets in terms of superfields to compare with the results of the supermatrix model. Namely, we should construct a superfield formulation of the supermultiplets. Recall that for supergravity, we can construct superfield formulation by starting with the supercovariant derivative ∇_A, and then imposing constraints on the field strengths which are defined as the coefficients of the operators ∇_A and $\mathcal{O}_{ab}$ in the commutators of ∇_A,

$$[\nabla_A, \nabla_B\} = C_{AB}{}^C(z)\nabla_C + R_{AB}{}^{cd}(z)\mathcal{O}_{cd}. \tag{2.12}$$

The equations of motion for supergravity are expressed as a constraint on the torsion tensor. It seems that we can construct superfield formulation for supermultiplets of massless higher spin fields in the same way. We consider (2.7) as the supercovariant derivative with vector index. We postulate the existence of the supermatrices with spinor index,

$$\mathcal{A}_\alpha = i\nabla_\alpha + a_\alpha(z) + \frac{i}{2}\{b_\alpha{}^B(z), \nabla_B\} + \frac{i}{2}\{\omega_\alpha{}^{bc}(z), \mathcal{O}_{bc}\} + \frac{i^2}{2}\{e_\alpha{}^{BC}(z), \nabla_B\nabla_C\} + \cdots . \tag{2.13}$$

We can regard that the supermatrices (2.7) and (2.13) as the supercovariant derivative for massless higher spin fields. The field strengths are defined as the coefficients of the operators ∇_A and $\mathcal{O}_{ab}$ in the commutators of (2.7) and (2.13).

In the next section, we will construct superfield formulation for the free theory of a massless supermultiplet with spins (3, 5/2) using this supercovariant derivative. Then, we will compare the results with those of the supermatrix model.

3. Superfield formalism of massless supermultiplet (3,5/2)

Now let us construct a superfield formulation of the free $4D$, $\mathcal{N}=1$ supersymmetric field theory of a massless supermultiplet with spins $(3, 5/2)$ by starting from the supercovariant derivative (2.7) and (2.13). The construction is similar to supergravity: we should impose constraints on superfields. One difference is in fixing gauge symmetries which act on auxiliary fields to eliminate auxiliary component fields. As in the case of supergravity we can construct on-shell and off-shell formulations. In this section we restrict attention to on-shell formulation. We give the results of off-shell formulation in appendix C.

Before we begin our analysis, we review some facts about the component formalism of massless higher spin gauge fields [5]. Totally symmetric tensor field of rank-s $\phi_{a_1\cdots a_s}(x)$ and tensor-spinor field of rank-$(s-1)$ $\psi_{a_1\cdots a_{s-1},\alpha}(x)$ are used to express massless boson and fermion system of a supermultiplet with spins $(s, s-\frac{1}{2})$.[5] We can construct the theory of $\phi_{a_1\cdots a_s}(x)$ and $\psi_{a_1\cdots a_{s-1}}(x)$ by requiring that the theory has the proper gauge symmetries. Let us postulate that the theory is invariant under the following gauge transformations:

$$\delta\phi_{a_1\cdots a_s}(x) = \partial_{(a_1}\lambda_{a_2\cdots a_s)}(x), \tag{3.1}$$

$$\delta\psi_{a_1\cdots a_{s-1},\alpha}(x) = \partial_{(a_1}\xi_{a_2\cdots a_{s-1}),\alpha}(x), \tag{3.2}$$

where the bracket () denotes symmetrization of the flat spacetime indices. The gauge parameters $\lambda_{a_1\cdots a_{s-1}}(x)$ and $\xi_{a_1\cdots a_{s-2},\alpha}(x)$ are rank-$(s-1)$ totally symmetric tensor with the traceless condition $\xi^b{}_{ba_1\cdots a_{s-3}} = 0$ and totally symmetric tensor-spinor with the gamma-traceless condition $(\gamma^b)_\alpha{}^\beta\xi_{ba_1\cdots a_{s-3},\beta} = 0$, respectively. If we impose the additional double traceless constraints $\phi''_{a_1\cdots a_{s-5}} = 0$ and triple-gamma traceless constraints $\not{\psi}'_{a_1\cdots a_{s-5}} = 0$, we can find that the gauge invariant free equations of motion for $\phi_{a_1\cdots a_s}(x)$ and $\psi_{a_1\cdots a_{s-1},\alpha}(x)$ are

$$W_{a_1\cdots a_s} \equiv \Box\phi_{a_1\cdots a_s} - s\partial_{(a_1}(\partial\cdot\phi)_{a_2\cdots a_s)} + s(s-1)\partial_{(a_1}\partial_{a_2}\phi'_{a_3\cdots a_s)} = 0, \tag{3.3}$$

$$Q_{a_1\cdots a_{s-1},\alpha} \equiv (\not{\partial})_{\alpha\beta}\psi_{a_1\cdots a_{s-1},}{}^\beta - (s-1)\partial_{(a_1}\not{\psi}_{a_2\cdots a_{s-1}),\alpha} = 0, \tag{3.4}$$

where we use the notations $(\partial\cdot\phi)_{a_1\cdots a_{s-1}} = \partial_b\phi^b{}_{a_1\cdots a_{s-1}}$, $\phi'_{a_1\cdots a_{s-3}} = \phi^b{}_{ba_1\cdots a_{s-2}}$,$(\gamma^a)_{\alpha\beta}\partial_a = (\not{\partial})_{\alpha\beta}$ and $\not{\psi}_{a,\alpha} = (\gamma^b)_{\alpha\beta}\psi_{ba_1\cdots a_{s-2}}{}^\beta$.

The conventional formulations for free totally symmetric tensor and tensor-spinor gauge fields have been originally derived by Fronsdal [1] and Fang-Fronsdal [2], respectively.

[5]From this section, Latin letters run from 1 to 4 and denote flat spacetime vector indices, and Greek letters run from 1 to 4 and denote spinor indices.

3.1 On-shell formulation of supermultiplet $(3, 5/2)$

In order to deal with the supermultiplet (3, 5/2) in terms of superfield, we keep track of the second order homogeneous polynomial of the operators ∂_a, ∇_α and $\mathcal{O}_{ab}$ in $\mathcal{A}_A$:

$$\begin{aligned}\mathcal{A}_A = i\nabla_A + i^2 e_A{}^{CD}(z)\nabla_C\nabla_D + \frac{i^2}{2}\omega_A{}^{C,de}(z)(\nabla_C\mathcal{O}_{de} + \mathcal{O}_{de}\nabla_C) \\ + \frac{i^2}{2}\Omega_A{}^{cd,ef}(z)(\mathcal{O}_{cd}\mathcal{O}_{ef} + \mathcal{O}_{ef}\mathcal{O}_{cd}),\end{aligned} \tag{3.5}$$

where the supercovariant derivative in flat superspace is defined as

$$\nabla_a = \partial_a, \quad \nabla_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\gamma^a)_{\alpha\beta}\theta^\beta\partial_a. \tag{3.6}$$

The commutation relations of the operators are given by

$$[\partial_a, \partial_b] = 0, \qquad [\partial_a, \nabla_\alpha] = 0, \qquad \{\nabla_\alpha, \nabla_\beta\} = 2i(\gamma^a)_{\alpha\beta}\nabla_a, \tag{3.7}$$

$$[\mathcal{O}_{ab}, \partial_c] = \frac{1}{2}(\delta_{ac}\partial_b - \delta_{bc}\partial_a), \qquad [\mathcal{O}_{ab}, \nabla_\alpha] = (\gamma_{ab})_\alpha{}^\beta\nabla_\beta. \tag{3.8}$$

In this and the next subsection, in order to deal with free field theories we keep only terms linear with respect to the component fields and use the flat supercovariant derivatives defined above.

The dynamical fields which describe the supermultiplet (3, 5/2) are expressed as

$$\phi_{abc}(x) = \frac{1}{3}\big(e_{a,bc}(z) + e_{b,ca}(z) + e_{c,ab}(z)\big)\Big|_{\theta=0}, \tag{3.9}$$

$$\psi_{ab,\alpha}(x) = \frac{1}{2}\big(e_{a,b\alpha}(z) + e_{b,a\alpha}(z)\big)\Big|_{\theta=0}, \tag{3.10}$$

where $e_{a,bc}(z)$ are the coefficients of $\partial_b\partial_c$ and $e_{a,b\alpha}(z)$ are the coefficients of $\partial_b\nabla_\alpha$ in (3.5). As we will see, these relations can be understood by looking at the gauge transformation properties of these fields. Local superfields appearing in (3.5) have too many unphysical degrees of freedom to describe the physical system of the massless supermultiplet (3, 5/2). Thus, in order to construct superfield formulation we must eliminate all the unphysical degrees of freedom. This is implemented by carrying out the following two procedures

- Imposing constraints on the superfields.
- Fixing the gauge symmetries.

We will perform these procedures in order.

3.1.1 Constraints

There are three types of constraints.[6]

[6]The constraints we impose in this subsection are summarized in appendix A. The superspace Bianchi identities subject to the on-shell constraints are given in appendix B.

1. The first type of constraints are summarized as follows:

$$e_{a,b}{}^{b}(z) = 0, \tag{3.11}$$

$$(\gamma^b)_\alpha{}^\beta e_{a,b\beta}(z) = 0. \tag{3.12}$$

As we will see later, we find the gauge transformation laws for $e_{a,bc}$: $\delta e_{a,bc} = \partial_a \lambda_{bc}$, and for $e_{a,b\beta}$: $\delta e_{a,b\beta} = \partial_a \xi_{b\beta}$. Thus, these constraints are necessary to be consistent with the traceless constraints on the gauge parameters $\lambda_b{}^b = 0$ and $(\gamma^b)_\alpha{}^\beta \xi_{b,\beta} = 0$.

2. The second type of constraints are imposed on the field strengths. Field strengths are the coefficients of the operators in the commutators of $\mathcal{A}_a$ and $\mathcal{A}_\alpha$, whose general expressions are given by the following forms:

$$\begin{aligned}[\mathcal{A}_A, \mathcal{A}_B] = & -iC_{AB}{}^{CD}(z)\nabla_C\nabla_D \\ & -\frac{i}{2}R_{AB}{}^{C,de}(z)(\nabla_C\mathcal{O}_{de} + \mathcal{O}_{de}\nabla_C) \\ & -\frac{i}{2}F_{AB}{}^{cd,ef}(z)(\mathcal{O}_{cd}\mathcal{O}_{ef} + \mathcal{O}_{ef}\mathcal{O}_{cd}).\end{aligned} \tag{3.13}$$

$C_{AB,CD}(z)$ are similar to the torsion tensor in supergravities because they include the first order derivatives of the vielbein fields $e_{a,bc}$ and $e_{a,b\alpha}$ with respect to x. $R_{AB,C,de}(z)$ are similar to the curvature tensor because they include the first order derivatives of the connection $\omega_{A,B,cd}$ with respect to x.[7] $F_{AB,cd,ef}(z)$ have no analogy in supergravities because they appear only for spin larger than 2.

We choose the following constraints:

$$C_{ab,}{}^{cd} = C_{ab}{}^{\gamma\delta} = 0, \qquad C_{a\alpha,}{}^{cd} = C_{a\alpha,}{}^{\gamma\delta} = 0, \tag{3.14}$$

$$C_{\alpha\beta,}{}^{cd} = 2i(\gamma^a)_{\alpha\beta}e_a{}^{cd}, \qquad C_{\alpha\beta,}{}^{c\gamma} = 2i(\gamma^a)_{\alpha\beta}e_a{}^{c\gamma}, \qquad C_{\alpha\beta,}{}^{\gamma\delta} = 2i(\gamma^a)_{\alpha\beta}e_a{}^{\gamma\delta}, \tag{3.15}$$

$$R_{ab}{}^{\gamma,cd} = 0, \qquad R_{a\alpha}{}^{\gamma,cd} = 0, \qquad R_{\alpha\beta}{}^{\gamma,cd} = 2i(\gamma^a)_{\alpha\beta}\omega_{a,}{}^{\gamma,cd}, \tag{3.16}$$

$$F_{ab}{}^{cd,ef} = 0, \qquad F_{a\alpha}{}^{cd,ef} = 0, \qquad F_{\alpha\beta}{}^{cd,ef} = 2i(\gamma^a)_{\alpha\beta}\Omega_{a,}{}^{cd,ef}. \tag{3.17}$$

The equations of motion for the supermultiplet can be expressed as the constraints on the field strength:

$$C_{a\alpha}{}^{c\gamma} = 0. \tag{3.18}$$

3. The third type of constraints are imposed for the equations of motion to be symmetric under permutation of the vector indices. The constraint $F_{ab,cd,ef} = 0$ implies that $\Omega_{a,bc,de}(z)$ can be written as a pure gauge like configuration

$$\Omega_{a,bc,de}(z) = \partial_a \chi_{bc,de}(z), \tag{3.19}$$

[7] In analogy with superfield formulations of supergravities, we can regard that the fields $e_{A,BC}(z)$ and $\omega_{A,B,cd}(z)$ are higher spin generalization of supervielbein and superspin connection.

where the parameter $\chi_{bc,de}$ satisfies $\chi_{bc,de} = -\chi_{cb,de} = -\chi_{bc,ed}$.[8] In order to make the equation of motion for the spin-3 field to be symmetric under permutations of the vector indices, we should impose

$$\chi_{bc,de}(z) = -\frac{1}{3}\omega_{[b,c],de}(z). \tag{3.20}$$

The constraint $R_{ab,\gamma,de} = 0$ implies that $\omega_{a,\gamma,cd}(z)$ can be written as a pure gauge configuration

$$\omega_{a,\gamma,cd}(z) = \partial_a \eta_{cd,\gamma}(z), \tag{3.21}$$

where $\eta_{cd,\gamma}$ satisfies $\eta_{cd,\gamma} = -\eta_{dc,\gamma}$. In order to make the equation of motion for the spin-$\frac{5}{2}$ field to be symmetric under permutations of the vector indices, we should impose

$$\eta_{cd,\gamma}(z) = -e_{[c,d],\gamma}(z). \tag{3.22}$$

Imposing the constraints (3.11), (3.12), (3.14)–(3.18), (3.20) and (3.22), we obtain[9]

$$[\mathcal{A}_a, \mathcal{A}_b] = -iC_{ab,}{}^{c\gamma}(z)\partial_c\nabla_\gamma - \frac{i}{2}R_{ab,}{}^{c,de}(z)(\partial_c\mathcal{O}_{de} + \mathcal{O}_{de}\partial_c), \tag{3.23}$$

$$\begin{aligned} [\mathcal{A}_a, \mathcal{A}_\alpha] &= -\frac{i}{2}R_{a\alpha}{}^{c,de}(z)(\partial_c\mathcal{O}_{de} + \mathcal{O}_{de}\partial_c) \\ \{\mathcal{A}_\alpha, \mathcal{A}_\beta\} &= -2i(\gamma^a)_{\alpha\beta}\mathcal{A}_a. \end{aligned} \tag{3.24}$$

With all these constraints, using the superspace Bianchi identities we can show that the equations of motion for spin-$\frac{5}{2}$ field

$$(\gamma^a)_{\alpha\beta}C_{ab,c,}{}^{\beta} = 0, \tag{3.25}$$

and for spin-3 field

$$R_{ab,c,d}{}^{a} = 0, \tag{3.26}$$

are satisfied. These equations can be derived in the same way as in [16, 19].

So far, we have analyzed the elimination of the unphysical degrees of freedom by imposing constraints. We have found that the equation of motion for spin-3 field (3.26) is expressed in terms of the second order derivatives of $e_{a,bc}$ and the one for spin-$\frac{5}{2}$ field (3.25)

[8] $\Omega_{a,bc,de}$ are called extra fields in the Vasiliev theory [11]. In this theory, these fields have been expressed as the second order derivatives of dynamical fields by imposing appropriate constraints. Furthermore, these fields have been used to construct higher derivative interactions. In this paper, we have imposed the constraints (3.20) on $\Omega_{a,bc,de}$. As a consequence of the constraints, the extra fields have been expressed as the second order derivatives of dynamical fields. Thus, it seems that the roles of the field are equal to those in the Vasiliev theory. The only difference is that their theory has been formulated in AdS spacetime, while our theory has been done in flat spacetime.

[9] Imposing the constraints, the commutators of $\mathcal{A}_\alpha$ are written as $\{\mathcal{A}_\alpha, \mathcal{A}_\beta\} = -2i(\gamma^a)_{\alpha\beta}\mathcal{A}_a - i(\gamma^a)_{\alpha\beta}\omega_a{}^{\gamma,cd} - \frac{i}{2}R_{\alpha\beta}{}^{\gamma.cd}$. Now, we define $\tilde{R}_{\alpha\beta}{}^{\gamma,cd} \equiv R_{\alpha\beta}{}^{\gamma,cd} + 2(\gamma^a)_{\alpha\beta}\omega_a{}^{\gamma,cd}$. From the superspace Bianchi identity (B.10), we can find that $\tilde{R}_{\alpha\beta,\gamma,cd} = 0$, so $\{\mathcal{A}_\alpha, \mathcal{A}_\beta\} = -2i(\gamma^a)_{\alpha\beta}\mathcal{A}_a$.

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is expressed in terms of the first order derivatives of $e_{a,b\alpha}$ respectively. They are symmetric under permutations of the vector indices. However, these constraints are not enough to eliminate all the unphysical degrees of freedom. The equations (3.4) and (3.3) are expressed in terms of the totally symmetric tensor fields ϕ_{abc} and $\psi_{ab,\alpha}$, but the equations (3.25) and (3.26) are expressed in terms of $e_{a,bc}$ and $e_{a,b\alpha}$, which have parts that are not totally symmetric. Thus, we should eliminate these degrees of freedom in order to show that the lowest components of the equations (3.25) and (3.26) coincide with the equations (3.4) and (3.3), respectively. We will do these in the next subsubsection.

3.1.2 Gauge fixing

There are two kinds of gauge symmetries 1. dynamical gauge symmetries 2. auxiliary gauge symmetries. A dynamical gauge symmetry has an action on a dynamical gauge field defined in (3.9) and (3.10), while an auxiliary gauge symmetry does not act on any of the dynamical gauge fields. An auxiliary gauge symmetry generates shifts of auxiliary gauge fields that are not determined in terms of the dynamical gauge fields by solving the constraints. These undetermined components are exactly those which we have mentioned in the last part of the previous subsubsection. Thus, we should eliminate these degrees of freedom by fixing gauge symmetries. Recall that gauge symmetries are embedded in the superunitary symmetry of the supermatrix model (2.8). We summarize the gauge transformations as follows:

1. Dynamical gauge transformations

   - $\Lambda = \lambda^{ab}\partial_a\partial_b$ generates

   $$\delta e_{a,bc} = \partial_a\lambda_{bc}, \quad \delta(others) = 0, \tag{3.27}$$

   where the parameter λ_{ab} satisfies $\lambda_{ab} = \lambda_{ba}$.

   - $\Lambda = \xi^{a,\alpha}\partial_a\nabla_\alpha$ generates

   $$\delta e_{a,b,\alpha} = \partial_a\xi_{b,\alpha}, \quad \delta(others) = 0. \tag{3.28}$$

2. Auxiliary gauge transformations

   - $\Lambda = \tilde{\lambda}^{a,bc}(\partial_a\mathcal{O}_{bc} + \mathcal{O}_{bc}\partial_a)$ generates

   $$\delta e_{a,bc} = \tilde{\lambda}_{b,ac} + \tilde{\lambda}_{c,ab}, \quad \delta\omega_{a,b,cd} = \partial_a\tilde{\lambda}_{b,cd}, \quad \delta(others) = 0, \tag{3.29}$$

   where $\tilde{\lambda}_{a,bc}$ satisfies $\tilde{\lambda}_{a,bc} = -\tilde{\lambda}_{a,cb}$.

   - $\Lambda = \tilde{\lambda}^{ab,cd}(\mathcal{O}_{ab}\mathcal{O}_{cd} + \mathcal{O}_{cd}\mathcal{O}_{ab})$ generates

   $$\delta\omega_{a,b,cd} = \tilde{\lambda}_{ab,cd} + \tilde{\lambda}_{cd,ab}, \quad \delta\Omega_{a,bc,de} = \partial_a\tilde{\lambda}_{bc,de}, \quad \delta(others) = 0, \tag{3.30}$$

   where $\tilde{\lambda}_{ab,cd}$ satisfies $\tilde{\lambda}_{ab,cd} = -\tilde{\lambda}_{ba,cd} = -\tilde{\lambda}_{ab,dc}$.

- $\Lambda = \tilde{\xi}^{ab,\alpha}(\mathcal{O}_{ab}\nabla_\alpha + \nabla_\alpha \mathcal{O}_{ab})$ generates

$$\delta e_{a,b,\alpha} = \tilde{\xi}_{ab,\alpha}, \quad \delta\omega_{a,\alpha,cd} = \partial_a \tilde{\xi}_{cd,\alpha}, \quad \delta(others) = 0, \tag{3.31}$$

  where $\tilde{\xi}_{ab,\alpha}$ satisfies $\tilde{\xi}_{ab,\alpha} = -\tilde{\xi}_{ba,\alpha}$.

Under the dynamical gauge transformations (3.27) and (3.28), the rank-3 totally symmetric tensor field $\phi_{abc}(x)$ defined in (3.9) and rank-2 totally symmetric tensor-spinor field $\psi_{ab,\alpha}(x)$ defined in (3.10) transform as follows :

$$\delta\phi_{abc}(x) = \partial_a\lambda_{bc}(x) + \partial_b\lambda_{ca}(x) + \partial_c\lambda_{ab}(x), \tag{3.32}$$

$$\delta\psi_{ab,\alpha}(x) = \partial_a\xi_{b,\alpha}(x) + \partial_b\xi_{a,\alpha}(x). \tag{3.33}$$

These correspond to (3.1) and (3.2), respectively. They are consistent with the identifications (3.9) and (3.10).

As we will now show, using the auxiliary gauge transformations (3.29), (3.30) and (3.31), we can eliminate the parts of $e_{a,bc}$ and $e_{a,b\alpha}$ that are not totally symmetric in the vector indices, and we can express dynamical variable in terms of ϕ_{abc} and $\psi_{ab,\alpha}$. We first fix the gauge symmetry (3.31). Gauge fixing can be done by transforming $e_{a,b\alpha} \rightarrow \hat{e}_{a,b\alpha} = e_{a,b\alpha} + \tilde{\xi}_{ab,\alpha}$, with choosing the parameter $\tilde{\xi}_{ab,\alpha}$ as

$$\tilde{\xi}_{ab,\alpha} = -e_{a,b,\alpha} + \psi_{ab,\alpha} + \frac{1}{2}(\gamma_a)_{\alpha\beta}\psi_{b,}{}^{\beta} - \frac{1}{2}(\gamma_b)_{\alpha\beta}\psi_{a,}{}^{\beta} - \frac{1}{3}\gamma_{ab}\psi'_\alpha \tag{3.34}$$

Carrying out this transformation, we can remove the part of $e_{a,b\alpha}$ that is not totally symmetric in the vector indices. Substituting $\hat{e}_{a,b\alpha}$ into the equation (3.25) we can show that the equation

$$(\gamma^a)_{\alpha\beta}C_{ab,c}{}^{\beta} = 0 \tag{3.35}$$

coincides with (3.4). Next, we fix the gauge symmetries (3.29) and (3.30). Gauge fixing can be done by transforming $e_{a,bc} \rightarrow \varepsilon_{a,bc} = e_{a,bc} + \tilde{\lambda}_{b,ac} + \tilde{\lambda}_{c,ab}$ and $\omega_{a,b,cd} \rightarrow w_{a,b,cd} = \omega_{a,b,cd} + \tilde{\lambda}_{ab,cd} + \tilde{\lambda}_{cd,ab}$, by choosing the parameters $\tilde{\lambda}_{a,bc}$ and $\tilde{\lambda}_{ab,cd}$ as we did in [14]. In [14], carrying out the gauge transformation by those parameters, we have removed the part of $e_{a,bc}$ that is not totally symmetric in the vector indices, and have shown that the equation

$$R_{ab,c,d}{}^{a} = 0 \tag{3.36}$$

coincides with (3.3). Thus, we have shown that with all these constraints and gauge fixing, we obtain the free theory of the supermultiplet (3, 5/2).

Before we close this section, we comment on the generalization of what we have done to a massless supermultiplet with spins $(s, s-\frac{1}{2})$. In order to deal with the supermultiplet, we keep track of the $(s-1)$th order polynomials of the operators in ∇_A and $\mathcal{O}_{ab}$:

$$\begin{aligned}\mathcal{A}_A = & \, i\nabla_A + (i)^{s-1} e_{A,}{}^{A_1\cdots A_{s-1}}(z)\nabla_{A_1}\cdots\nabla_{A_{s-1}} \\ & +\frac{(i)^{s-1}}{s-1}\omega_{A,}{}^{A_1\cdots A_{s-2},b_1c_1}(z)\{\nabla_{A_1}\cdots\nabla_{A_{s-2}}\mathcal{O}_{b_1c_1}\} \\ & +\frac{(i)^{s-1}}{(s-1)(s-2)}\Omega_{A,}{}^{A_1\cdots A_{s-3},b_1c_1,b_2c_2}(z)\{\nabla_{A_1}\cdots\nabla_{A_{s-3}}\mathcal{O}_{b_1c_1}\mathcal{O}_{b_2c_2}\} \\ & +\frac{(i)^{s-1}}{(s-1)(s-2)(s-3)}\tilde{\Omega}_{(1),A,}{}^{A_1\cdots A_{s-4},b_1c_1,b_2c_2,b_3c_3}(z)\{\nabla_{A_1}\cdots\nabla_{A_{s-4}}\mathcal{O}_{b_1c_1}\mathcal{O}_{b_2c_2}\mathcal{O}_{b_3c_3}\} \\ & \quad \vdots \\ & +\frac{(i)^{s-1}}{(s-1)!}\tilde{\Omega}_{(s-3),A}{}^{b_1c_1,\ldots,b_{s-1}c_{s-1}}(z)\{\mathcal{O}_{b_1c_1}\cdots\mathcal{O}_{b_{s-1}c_{s-1}}\}.\end{aligned} \tag{3.37}$$

From the discussion in this section, it seems that $\tilde{\Omega}_{(i)}(z)(i=1,\ldots,s-3)$ are not necessary to construct a superfield formulation of the supermultiplet $(s, s-\frac{1}{2})$. We set these auxiliary fields to zero: $\tilde{\Omega}_1(z) = \cdots = \tilde{\Omega}_{s-3}(z) = 0$. Starting from this $\mathcal{A}_A$ we may construct a superfield formulation of the massless supermultiplet $(s, s-\frac{1}{2})$ using the same method as the one we have employed in this section.

4. Supermatrix model

Now, with the superfield formulation of the massless supermultiplet (3, 5/2), we can compare the results with those of the supermatrix model. Imposing the constraints in section 3, we obtain the equations of motion of the supermatrix model

$$\begin{aligned}[\mathcal{A}^a,[\mathcal{A}_a,\mathcal{A}_b]] &= [\partial^a, C_{ab,}{}^{c,\gamma}\partial_c\nabla_\gamma + R_{ab}{}^{c,de}(\partial_c\mathcal{O}_{de}+\mathcal{O}_{de}\partial_c)] \\ &= (\partial^a C_{ab}{}^{c\gamma})\partial_c\nabla_\gamma + (R_{ab,}{}^{c,da})\partial_c\partial_d + (\partial^a R_{ab,}{}^{c,de})(\partial_c\mathcal{O}_{de}+\mathcal{O}_{de}\partial_c) = 0\end{aligned} \tag{4.1}$$

The equation $R_{ab,}{}^{c,da} = 0$ coincides with (3.26). $\partial^a R_{ab,}{}^{c,de} = 0$ follows from the superspace Bianchi identity (B.3) by contracting a and e. $\partial^a C_{ab,}{}^{c\gamma} = 0$ is obtained by multiplying $(\gamma^a)^{\gamma\alpha}\nabla_\gamma$ to (B.5). Therefore, we have shown that solutions of the equations of motion for the massless supermultiplet (3, 5/2) satisfy the equations of motion of the supermatrix model (4.1).

5. Conclusions and future works

In this paper, we have studied the relation between a supermatrix model and the free $4D$, $\mathcal{N}=1$ supersymmetric field theory of a massless supermultiplet with spins (3, 5/2) on the basis of [16]. In order to do this, we have constructed a superfield formulation of the supermultiplet. Then, we have shown that solutions of the equations of motion for the supermultiplet satisfy the equations of motion of the supermatrix model. It is difficult to show the converse that is to derive the equations of motion for the supermultiplet from the equations of motion of the supermatrix model. We may generalize what we have done in this paper to the supermultiplet $(s, s-\frac{1}{2})$ using the same method as the one we have employed.

A superfield formulation of massless higher spin gauge theory in four-dimensional AdS spacetime has been constructed in [22]. In [22], the authors have given the on-shell superspace constraints. They have also shown that the linearized constraints describe the free equations of motion of higher spin fields with cosmological constant. In the case of the supermultiplet (3, 5/2), it seems that the linearized constraints are equal to those we have imposed. The only difference is that we have formulated in flat spacetime, while they have done in AdS spacetime. Their theory may be useful to construct interacting supersymmetric higher spin gauge theories by matrix models.

There are several things which should be studied further. One is to investigate the tensor fields which are not totally symmetric in the spacetime vector indices can be included in matrix models. Viewed from matrix models, field strengths should be introduced as independent degrees of freedom. There is a possibility that "field strengths" propagate as tensor fields that are not totally symmetric. This possibility has been studied in [23]. The authors have investigated that the fields appear as the coefficients of terms linear in the covariant derivative and local Lorentz generators. They have found that some components of the torsion can be identified with a scalar and rank-2 antisymmetric tensor field, and have shown that the equations of these fields can be derived from that of a matrix model. It is interesting to extend this analysis to the fields that appear as coefficients of higher order terms in covariant derivative and local Lorentz generators.

Another thing to be pursue is to construct the interacting massless higher spin gauge field theory. Difficulties associated with the requirement of gauge invariance can be overcome by using the matrix model because it has gauge invariance manifestly.

Acknowledgments

I am grateful to Y. Baba, N. Hatano, N. Ishibashi, K. Murakami, Y. Satoh and K. Yamamoto for useful discussions.

A. Summary of constraints

1. Constraints on component fields:

$$e_{a,b}{}^{b} = 0, \tag{A.1}$$

$$(\gamma^b)_\alpha{}^\beta e_{a,b,\beta} = 0. \tag{A.2}$$

2. Constraints on superfield strengths

   Off-shell constraints:

$$C_{ab,}{}^{cd} = C_{ab,}{}^{\gamma\delta} = 0, \qquad C_{a\alpha,}{}^{cd} = C_{a\alpha,}{}^{\gamma\delta} = 0, \tag{A.3}$$

$$C_{\alpha\beta,}{}^{cd} = 2i(\gamma^a)_{\alpha\beta} e_{a,}{}^{cd}, \qquad C_{\alpha\beta,}{}^{c\gamma} = 2i(\gamma^a)_{\alpha\beta} e_{a,}{}^{c\gamma}, \qquad C_{\alpha\beta,}{}^{\gamma\delta} = 2i(\gamma^a)_{\alpha\beta} e_{a,}{}^{\gamma\delta}. \tag{A.4}$$

$$R_{ab,}{}^{\gamma,cd} = R_{a\alpha}{}^{\gamma,cd} = 0, \qquad R_{\alpha\beta}{}^{\gamma,cd} = 2i(\gamma^a)_{\alpha\beta}\omega_{a,}{}^{\gamma,cd}. \tag{A.5}$$

$$F_{ab,}{}^{cd,ef} = F_{a\alpha}{}^{cd,ef} = 0, \qquad F_{\alpha\beta}{}^{cd,ef} = 2i(\gamma^a)_{\alpha\beta}\Omega_{a,}{}^{cd,ef}. \tag{A.6}$$

On-shell constraints:

$$C_{a\alpha}{}^{c\gamma} = 0. \tag{A.7}$$

3. Constraints on "pure gauge" field:

$$\chi_{bc,de} = -\frac{1}{3}\omega_{[bc],de}, \tag{A.8}$$

$$\eta_{cd,\gamma} = -e_{[c,d],\gamma}. \tag{A.9}$$

B. Bianchi identities

We give the superspace Bianchi identities subject to the constraints (A.1), (A.2), (A.3), (A.4), (A.5), (A.6), (A.7), (A.8) and (A.9).

1. $[\mathcal{A}_a, [\mathcal{A}_b, \mathcal{A}_c]] + [\mathcal{A}_b, [\mathcal{A}_c, \mathcal{A}_a]] + [\mathcal{A}_c, [\mathcal{A}_a, \mathcal{A}_b]] = 0$ gives

$$\partial_{[a} C_{bc]}{}^{d\delta} = 0, \tag{B.1}$$

$$R_{[ab,}{}^{d,e}{}_{c]} = 0, \tag{B.2}$$

$$\partial_{[a} R_{bc]}{}^{d,ef} = 0. \tag{B.3}$$

2. $[\mathcal{A}_a, [\mathcal{A}_b, \mathcal{A}_\alpha]] + [\mathcal{A}_b, [\mathcal{A}_\alpha, \mathcal{A}_a]] + [\mathcal{A}_\alpha, [\mathcal{A}_a, \mathcal{A}_b]] = 0$ gives

$$2i(\gamma^d)_{\alpha\beta} C_{ab,}{}^{c\beta} + \frac{1}{2} R_{b\alpha,}{}^{c,d}{}_a - \frac{1}{2} R_{a\alpha,}{}^{c,d}{}_b = 0, \tag{B.4}$$

$$\nabla_\alpha C_{ab}{}^{c\beta} + \frac{1}{2} R_{ab}{}^{c,de} (\gamma_{de})_\alpha{}^\beta = 0, \tag{B.5}$$

$$\nabla_\alpha R_{ab}{}^{c,de} + \partial_a R_{b\alpha,}{}^{c,de} - \partial_b R_{a\alpha,}{}^{c,de} = 0. \tag{B.6}$$

3. $[\mathcal{A}_a, \{\mathcal{A}_\alpha, \mathcal{A}_\beta\}] + \{\mathcal{A}_\alpha, [\mathcal{A}_\beta, \mathcal{A}_a]\} - \{\mathcal{A}_\beta, [\mathcal{A}_a, \mathcal{A}_\alpha]\} = 0$ gives

$$\tilde{R}_{\alpha\beta,}{}^{c,d}{}_e = 0, \tag{B.7}$$

$$2(\gamma^b)_{\alpha\beta} C_{ab,}{}^{c\gamma} + \frac{1}{2} R_{a\beta,}{}^{c,de} (\gamma_{de})_\alpha{}^\gamma + \frac{1}{2} R_{a\alpha,}{}^{c,de} (\gamma_{de})_\beta{}^\gamma = 0, \tag{B.8}$$

$$2(\gamma^b)_{\alpha\beta} R_{ab}{}^{c,de} + \nabla_\alpha R_{a\beta,}{}^{c,de} + \nabla_\beta R_{a\alpha,}{}^{c,de} = 0. \tag{B.9}$$

4. $[\mathcal{A}_\alpha, \{\mathcal{A}_\beta, \mathcal{A}_\gamma\}] + [\mathcal{A}_\beta, \{\mathcal{A}_\gamma, \mathcal{A}_\alpha\}] + [\mathcal{A}_\gamma, \{\mathcal{A}_\alpha, \mathcal{A}_\beta\}] = 0$ gives

$$\tilde{R}_{(\alpha\beta}{}^{c,de} (\gamma_{de})_{\gamma)}{}^\delta = 0, \tag{B.10}$$

$$2(\gamma^a)_{(\alpha\beta} R_{a\gamma)}{}^{c,de} + \nabla_{(\alpha} \tilde{R}_{\beta\gamma)}{}^{c,de} = 0. \tag{B.11}$$

C. Solution of the Bianchi identities

In this appendix, we give the results of an off-shell superfield formulation for the theory of a massless supermultiplet with spins (3, 5/2). To construct the formalism, we should impose the off-shell constraints which reduce the number of components, and solve Bianchi identities subject to the off-shell constraints. We can find that the Bianchi identities reduce the number of independent superfields to one complex vector field R_a, one real symmetric-traceless tensor G_{ab} and one chiral superfield $W_{a,\alpha\beta\gamma}$. We can find explicit expressions for the superfield strengths in terms of these superfields.

As in the case of supergravity, the lowest components of R_a and G_{ab} with respect to θ are physical degrees of freedom. The counting of field components are as follows:

Bosonic	$\phi_{abc}(x)$	$\lambda_{ab}(x)$	$R_a(x)$	$G_{ab}(x)$	
	$+20$	-9	$+8$	$+9$	$=28$
Fermionic	$\psi_{ab,\alpha}(x)$	$\xi_{a,\alpha}(x)$			
	$+40$	-12			$=28$

Therefore, the number of bosonic and fermionic degrees of freedom are equal.

Here we use a two spinor notation of [20]. The coordinates of flat superspace are denoted by $z^A = (x^a, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$. Latin indices a denote Lorentz tensor indices, Greek indices $(\alpha, \dot{\alpha})$ denote spinor indices. Covariant derivatives in flat superspace are defined as follows

$$\nabla_a = \partial_a, \tag{C.1}$$

$$\nabla_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^a{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_a, \tag{C.2}$$

$$\overline{\nabla}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^a{}_{\alpha\dot{\alpha}}\partial_a. \tag{C.3}$$

We list the results of the off-shell constraints and the solution of superspace Bianchi identities.

Constraints. We impose constraints on superfield strengths. The commutators of $\mathcal{A}_A$ can be written as follows:

$$\begin{aligned}[\mathcal{A}_A, \mathcal{A}_B\} = & -iC_{AB}{}^{CD}\nabla_C\nabla_D - \frac{i}{2}R_{AB}{}^{D,ef}(\nabla_D\mathcal{O}_{ef} + \mathcal{O}_{ef}\nabla_D) \\ & -\frac{i}{2}F_{AB}{}^{cd,ef}(\mathcal{O}_{cd}\mathcal{O}_{ef} + \mathcal{O}_{ef}\mathcal{O}_{cd}).\end{aligned} \tag{C.4}$$

We choose the following constraints on superfield strengths:

$$C_{ab,cd} = C_{ab,\gamma\delta} = C_{ab,\gamma\dot{\delta}} = 0, \qquad C_{a\alpha,cd} = C_{a\alpha,\gamma\delta} = C_{a\alpha,\dot{\gamma}\dot{\delta}} = C_{a\alpha,\gamma\dot{\delta}} = 0 \tag{C.5}$$

$$C_{\alpha\beta,cd} = C_{\alpha\beta,d\delta} = C_{\alpha\beta,d\dot{\delta}} = C_{\alpha\beta,\dot{\gamma}\dot{\delta}} = C_{\alpha\beta,\gamma\dot{\delta}} = C_{\alpha\beta,\gamma\delta} = 0 \tag{C.6}$$

$$C_{\alpha\dot{\alpha},cd} = 2i(\sigma^a)_{\alpha\dot{\alpha}}e_{a,cd}, \quad C_{\alpha\dot{\alpha},d\delta} = 2i(\sigma^a)_{\alpha\dot{\alpha}}e_{a,d\delta}, \tag{C.7}$$

$$C_{\alpha\dot{\alpha},\gamma\delta} = 2i(\sigma^a)_{\alpha\dot{\alpha}}e_{a,\gamma\delta}, \quad C_{\alpha\dot{\alpha},\delta\dot{\delta}} = 2i(\sigma^a)_{\alpha\dot{\alpha}}e_{a,\delta\dot{\delta}}. \tag{C.8}$$

$$R_{ab,\delta,cd} = 0, \qquad R_{a\alpha,\delta,cd} = R_{a\alpha,\dot{\delta},cd} = 0, \qquad R_{\alpha\beta,\delta,cd} = R_{\alpha\beta,\dot{\delta},cd} = 0, \tag{C.9}$$

$$R_{\alpha\dot{\alpha},\delta,cd} = 2i(\sigma^a)_{\alpha\dot{\alpha}}\omega_{a,\delta,cd}, \tag{C.10}$$

$$F_{ab,cd,ef} = 0, \qquad F_{a\alpha,cd,ef} = 0, \qquad F_{\alpha\beta,cd,ef} = 2i(\sigma^a)_{\alpha\dot{\alpha}}\Omega_{a,cd,ef}, \tag{C.11}$$

$$R_{\alpha\dot{\alpha},\delta}{}^{\dot{\delta}}{}_{,\beta\gamma} = 0, \qquad C_{\alpha\dot{\alpha}\beta,\delta}{}^{\dot{\alpha}}{}_{\gamma} = 0, \tag{C.12}$$

and their complex conjugates. Here, we define $R_{\alpha\dot{\alpha},\delta\dot{\delta},\beta\gamma} \equiv (\sigma^d)_{\delta\dot{\delta}}R_{\alpha\dot{\alpha},d,\beta\gamma}$ and $C_{\alpha\dot{\alpha}\beta,\delta\dot{\delta}\gamma} \equiv (\sigma^d)_{\delta\dot{\delta}}C_{\alpha\dot{\alpha}\beta,d\gamma}$.

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Solution of the Bianchi identities.

1. Constraints on the superfields W, G and R

$$\overline{\nabla}_{\dot\alpha}R_c = 0, \tag{C.13}$$
$$\nabla^\alpha G_{c,\alpha\dot\beta} = \nabla_{\dot\beta}R_c^\dagger, \tag{C.14}$$
$$\overline{\nabla}_{\dot\alpha}W_{c,\beta\gamma\delta} = 0, \tag{C.15}$$
$$\nabla^\alpha W_{c,\alpha\beta\delta} + \frac{i}{2}(\nabla_{\beta\dot\beta}G_{c,\delta}{}^{\dot\beta} + \nabla_{\delta\dot\beta}G_{c,\beta}{}^{\dot\beta}) = 0, \tag{C.16}$$
$$G^\dagger_{c,\alpha\dot\alpha} = G_{c,\alpha\dot\alpha}, \tag{C.17}$$
$$W^\dagger_{c,\alpha\beta\delta} = \overline{W}_{c,\dot\alpha\dot\beta\dot\delta}, \tag{C.18}$$
$$G_{\gamma\dot\gamma,\alpha\dot\alpha} = (\sigma^c)_{\gamma\dot\gamma}G_{c,\alpha\dot\alpha} = G_{(\gamma\alpha)(\dot\gamma\dot\alpha)}, \tag{C.19}$$

and their complex conjugates.

As a consequence of the constraints (C.12), $G_{a,b}$ has only the traceless symmetric components.

2. Torsion

$$C_{\alpha\dot\alpha\dot\beta,c\delta} = (\sigma^a)_{\alpha\dot\alpha}C_{a\dot\beta,c\delta} = -2i\epsilon_{\dot\beta\dot\alpha}\epsilon_{\alpha\delta}R_c, \tag{C.20}$$
$$C_{\alpha\dot\alpha\beta,c\delta} = (\sigma^a)_{\alpha\dot\alpha}C_{a\beta,c\delta} = \frac{i}{4}(\epsilon_{\alpha\delta}G_{c,\beta\dot\alpha} - 3\epsilon_{\beta\alpha}G_{\delta\dot\alpha} - 3\epsilon_{\beta\delta}G_{c,\alpha\dot\alpha}), \tag{C.21}$$
$$\begin{aligned} C_{\alpha\dot\alpha\beta\dot\beta,c\dot\delta} &= -2\epsilon_{\alpha\beta}\overline{W}_{\dot\alpha\dot\beta\dot\delta,c} - \frac{1}{2}\epsilon_{\alpha\beta}(\epsilon_{\dot\delta\dot\beta}\nabla^\epsilon G_{c,\epsilon\dot\alpha} + \epsilon_{\dot\delta\dot\alpha}\nabla^\epsilon G_{c,\epsilon\dot\beta}) \\ &\quad + \frac{1}{2}\epsilon_{\dot\alpha\dot\beta}(\nabla_\alpha G_{c,\beta\dot\delta} + \nabla_\beta G_{c,\alpha\dot\delta}), \end{aligned} \tag{C.22}$$

and their complex conjugates.

3. Curvature

$$R_{\dot\alpha\dot\beta,c,\dot\delta\dot\epsilon} = 4(\epsilon_{\dot\alpha\dot\epsilon}\epsilon_{\dot\beta\dot\delta} + \epsilon_{\dot\beta\dot\epsilon}\epsilon_{\dot\alpha\dot\delta})R_c, \tag{C.23}$$
$$R_{\dot\alpha\dot\beta,c,\delta\epsilon} = 0, \tag{C.24}$$
$$R_{\alpha\dot\alpha,c,\beta\delta} = \epsilon_{\beta\alpha}G_{c,\delta\dot\alpha} + \epsilon_{\delta\alpha}G_{c,\beta\dot\alpha}, \tag{C.25}$$
$$\begin{aligned} R_{\alpha\dot\alpha\beta,c,\delta\epsilon} &= \frac{i}{2}(\epsilon_{\beta\alpha}\nabla_\delta + \epsilon_{\beta\delta}\nabla_\alpha)G_{c,\epsilon\dot\alpha} + \frac{i}{2}(\epsilon_{\beta\delta}\nabla_\epsilon + \epsilon_{\beta\epsilon}\nabla_\alpha)G_{c,\delta\dot\alpha} \\ &\quad + i(\epsilon_{\epsilon\beta}\epsilon_{\alpha\delta} + \epsilon_{\delta\beta}\epsilon_{\alpha\epsilon})\nabla^\zeta G_{c,\zeta\dot\alpha}, \end{aligned} \tag{C.26}$$
$$R_{\alpha\dot\alpha\beta,c,\dot\delta\dot\epsilon} = 4i\epsilon_{\beta\alpha}\overline{W}_{c,\dot\alpha\dot\delta\dot\epsilon} + \frac{i}{2}(\epsilon_{\dot\alpha\dot\delta}\nabla_\beta G_{c,\alpha\dot\epsilon} + \epsilon_{\dot\alpha\dot\epsilon}\nabla_\beta G_{c,\alpha\dot\delta}), \tag{C.27}$$
$$R_{\alpha\dot\alpha\beta\dot\beta,c,\delta\dot\delta\epsilon\dot\epsilon} = (\sigma^a)_{\alpha\dot\alpha}(\sigma^b)_{\beta\dot\beta}(\sigma^d)_{\delta\dot\delta}(\sigma^e)_{\epsilon\dot\epsilon}R_{ab,c,de} \tag{C.28}$$
$$\begin{aligned} &= 4\epsilon_{\alpha\beta}\epsilon_{\delta\epsilon}\overline{X}_{c,(\dot\alpha\dot\beta)(\dot\delta\dot\epsilon)} + 4\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\delta\dot\epsilon}X_{c,(\alpha\beta)(\delta\epsilon)} \\ &\quad -4\epsilon_{\alpha\beta}\epsilon_{\dot\delta\dot\epsilon}\overline{\Psi}_{c,(\dot\alpha\dot\beta)(\delta\epsilon)} - 4\epsilon_{\dot\alpha\dot\beta}\epsilon_{\delta\epsilon}\Psi_{c,(\alpha\beta)(\dot\delta\dot\epsilon)}, \end{aligned} \tag{C.29}$$
$$\Psi_{c,(\alpha\beta)(\dot\delta\dot\epsilon)} = \overline{\Psi}_{c,(\dot\delta\dot\epsilon)(\alpha\beta)}, \tag{C.30}$$

$$X_{c,(\alpha\beta)(\delta\epsilon)} = -\frac{1}{4}(\nabla_\alpha W_{c,\beta\delta\epsilon} + \nabla_\beta W_{c,\delta\epsilon\alpha} + \nabla_\delta W_{c,\epsilon\alpha\beta} + \nabla_\epsilon W_{c,\alpha\beta\delta}) + \frac{1}{16}(\overline{\nabla}_{\dot\zeta}\overline{\nabla}^{\dot\zeta} R_c^\dagger + \nabla^\zeta\nabla_\zeta R_c), \tag{C.31}$$

$$\Psi_{c,(\alpha\beta)(\dot\delta\dot\epsilon)} = \frac{i}{8}(\nabla_{\beta\dot\delta} G_{c,\alpha\dot\epsilon} + \nabla_{\alpha\dot\delta} G_{c,\beta\dot\epsilon} + \nabla_{\beta\dot\epsilon} G_{c,\alpha\dot\delta} + \nabla_{\alpha\dot\epsilon} G_{c,\beta\dot\delta}) + \frac{1}{8}(\overline{\nabla}_{\dot\delta}\nabla_\beta G_{c,\alpha\dot\epsilon} + \overline{\nabla}_{\dot\delta}\nabla_\alpha G_{c,\beta\dot\epsilon} + \overline{\nabla}_{\dot\epsilon}\nabla_\beta G_{c,\alpha\dot\delta} + \overline{\nabla}_{\dot\epsilon}\nabla_\alpha G_{c,\beta\dot\delta}), \tag{C.32}$$

and their complex conjugates.

All other components vanish.

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